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Problem-Based Learning

Professional Development
Participant Guide

ALWAYS LEARNING PEARSON

Pearson North America School Services
enVisionmath**2.0 Common Core © 2016**
Problem-Based Learning
Professional Development Workshop
Participant Guide

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Table of Contents

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Agenda

Introduction
Module 1: Introduction to Problem-Based Instruction
Module 2: Problem-Based Learning in **enVision**math**2.0**
Module 3: Supporting Students' Learning through Assessment and Differentiated Instruction
Module 4: Planning for Problem-Based Learning
Reflection and Closing

Outcomes

At the conclusion of this workshop, you will be able to:

- Incorporate Problem-Based Learning strategies within the **enVision**math**2.0** Common Core classroom
- Describe strategies to engage a range of learners through lesson modification and activity-based instruction
- Implement an interactive instructional model that supports in-depth conceptual development

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Module 1: Introduction to Problem-Based Instruction

Big Questions

What does a problem-based approach to mathematics instruction entail?

What are the stages of a problem-based lesson?

Reflection

Consider the following statement about the meaning of a problem. What stands out to you about this statement?

"A *problem* involves a situation in which a person wants something and does not know immediately what to do to get it." (Reys, et al., 1998, p. 70)

Reflect on your own mathematical learning. Which sentence or sentences best describe the messages that you received during your years as a mathematics learner?

1. Arriving at the correct answer to a problem is the most important thing.
2. The methods that you use to solve a problem are just as important as arriving at the correct solution.
3. The teacher is responsible for telling students the best way to solve a problem, and students must follow that procedure for full credit.
4. There are many ways that you can solve a problem that will ultimately lead to the correct solution. It is the students' responsibility to pursue a variety of solution methods and to justify their reasoning.

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Student Strategies

Primary

Monica has 24 crayons. Paul has 64 crayons. How many crayons do they have in all? Solve this problem any way you choose. Explain your work.

Intermediate

Kevin's family took 239 photos on their summer vacation. Marco and his family took 12 times as many photos on their vacation. How many photos did Marco's family take? Solve this problem any way you choose. Explain your work.

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Teaching through Problems

"The single most important principle for improving the teaching of mathematics is to *allow the subject of mathematics to be problematic for students* (Hiebert et al., 1996). That is, students solve problems not to apply mathematics but also to learn new mathematics. When students engage in well-chosen problem-based tasks and focus on the solution methods, what results is new understanding of the mathematics embedded in the task. When students are actively looking for relationships, analyzing patterns, finding out which methods work and which don't, justifying results, or revaluating and challenging the thoughts of others, they are necessarily and optimally engaging in reflective thought about the ideas involved. The appropriate dots in their cognitive structure are acting to give meaning to new ideas. *Most, if not all, important mathematics concepts and procedures can best be taught through problem solving.*"

—Van de Walle and Lovin (2006, 11)

Features of a Problem

Some problems provide more opportunity for students to develop understanding than others. The information below from *Teaching Student-Centered Mathematics* (Van de Walle and Lovin 2006, 11) outlines three features of rich math problems.

- *The problem must begin where the students are.* The design or selection of the task should take into consideration the current understanding of the students. They should have the appropriate ideas to engage and solve the problem and yet still find it challenging and interesting. In other words, it should be within their zone of proximal development.
- *The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.* In solving the problem or doing the activity, students should be concerned primarily with making sense of the mathematics involved and thereby developing their understanding of those ideas. Although it is acceptable and even desirable to have contexts or external conditions for problems that make them interesting, these aspects should not overshadow the mathematics to be learned.
- *The problem must require justifications and explanations for answers and methods.* Students should understand that the responsibility for determining if answers are correct and why rests with them. Students should also expect to explain their solution methods as a natural part of solving problems.

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Three-Stage Format

Before	
During	
After	

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Module 2: Problem-Based Learning in enVisionmath2.0

Big Questions

What are the Standards for Mathematical Practice?

How does a problem-based approach promote the mathematical practices?

What are the benefits of using questioning during PBL?

A Closer Look at the Mathematical Practices

Watch and Respond

Jane Schielack on the Mathematical Practices
Associate Dean for Assessment and Pre-K–12 Education
Texas A&M University

Use the space below to take notes as you watch the video.

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Categorizing the Mathematical Practices

May Seem Familiar	Might Be Easily Misinterpreted	Sounds Unfamiliar

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Problem-Based Learning

Watch and Respond

Janet Caldwell on Problem-Based Learning

Professor of Mathematics
Rowan University

What is PBL?

What are the students' roles in PBL?

What is the teacher's role in PBL?

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A Classroom Environment that Supports PBL and the Mathematical Practices

Brainstorm characteristics of a classroom environment that supports Problem-Based Learning and students' development of the mathematical practices. Consider the following:

- What types of tasks are students engaged in?
- What is the role of the teacher?
- What is the role of each student?
- What is the social culture of the classroom?
- What mathematical tools are available?
- How is learning accessible to all students?

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Critical Features of Classrooms that Support PBL and the Mathematical Practices

Dimensions of Classrooms	Core Features of Classrooms
Nature of classroom tasks	Make mathematics problematic. Connect with where students are. Leave behind something of mathematical value.
Role of the teacher	Select tasks with goals in mind. Share essential information. Establish classroom culture.
Social culture of the classroom	Ideas and methods are valued. Students choose and share their methods. Mistakes are learning sites for everyone. Correctness resides in mathematical argument.
Mathematical tools as Learning Supports	Meaning for tools must be constructed by the user. Use tools with a purpose—to solve problems. Use tools for recording, communicating, and thinking.
Equity and accessibility	Tasks are accessible to all students. Every student is heard. Every student contributes.

(Hiebert et al. 1997, 12)

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Before, During, and After Card Sort

Use the graphic organizer below to take notes on what activities and actions occur during each portion of the PBL lesson format in **enVision**math**2.0**.

Before	During	After

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Activity Observations

Standards for Mathematical Practice Check the Standards for Mathematical Practice that you observed during the lesson. Be prepared to provide evidence.	**Teacher Questioning** In the space below, write down every question that the teacher asks as he or she teaches the lesson.
☐ Make sense of problems and persevere in solving them. ☐ Reason abstractly and quantitatively. ☐ Construct viable arguments and critique the reasoning of others. ☐ Model with mathematics. ☐ Use appropriate tools strategically. ☐ Attend to precision.	

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☐ Look for and make use of structure. ☐ Look for and express regularity in repeated reasoning.	

(NGA Center and CCSSO 2010, 6–8)

Activity Debrief

Which mathematical practices did you experience or observe throughout the PBL activity? Provide evidence.

Review the list of questions that the teacher asked throughout the PBL activity. How might you categorize those questions?

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Teacher Questioning

Tips for Expanding Teacher Questioning

- Use students' answers as part of further probing questions.
- Stay focused on the Essential Understanding/Focus Question of the lesson and make sure your questioning throughout the lesson reflects this focus.
- Vary the level of complexity of the questions you ask, carefully selecting students to respond verbally to each question based on their grasp of mathematical concepts.
- Select a few question frames to use for a period of time to expand your own repertoire of questioning and extend students' thinking through a familiar routine.

(Coggins et al. 2007, 80–81)

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Question Frames

Question	Assess or Extend		Mathematical Practice(s)
How would you describe the problem in your own words?			
What information do you have? What do you need to find out?			
What strategies are you going to use?			
What tools will you need?			
Is there another tool you could use that might be more efficient?			
What is the advantage of using this tool?			
Can you draw a picture or make a model to show this?			
Can you represent what is happening in this problem with a number sentence?			
Can you represent what you have drawn on your paper using numbers?			
If you compare your work with anyone else's, what did they try?			
How did you get your answer?			
Can you describe your method?			
Can you explain why it works?			

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Question	Assess or Extend		Mathematical Practice(s)
Why did you decide to use this method?			
Can you think of another method that might have worked?			
Is there a more efficient strategy?			
Did you use or learn any new words today? What do they mean?			
Can you describe what you did using math vocabulary?			
Is that true for all cases? Explain.			
Can you think of a counterexample?			
How would you prove that?			
Do you think this would work with other numbers?			
Have you thought of all the possibilities? How can you be sure?			
What assumptions are you making?			
Can you explain what you have done so far? What else is there to do?			
What do you think about what__said?			
Do you agree? Why or why not?			

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Question	Assess or Extend		Mathematical Practice(s)
Does anyone have the same answer but a different way to explain it?			
Do you understand what_____is saying?			
Can you convince the rest of us that your answer makes sense?			
What would happen if…? What if not?			
Do you see a pattern? Can you explain the pattern?			

(PBS, 2002)

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Eight Mathematics Teaching Practices

Review the following eight mathematics teaching practices that have been found to have an impact on student learning and that should be a consistent feature of every mathematics lesson.

- **Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
- **Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
- **Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
- **Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
- **Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
- **Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
- **Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
- **Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

(NCTM, 2014, p. 3)

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Reflection and Next Steps

List the big ideas and specific ideas for implementation for each category from today's workshop. Place a check mark near the ones that you already practice regularly in your classroom. Circle at least three ideas that you plan to implement in your classroom.

	Problem-Based Instruction	**Assessing the Standards for Mathematical Practice**	**Strategies for Differentiating Instruction**
Big Ideas			
Ideas for Implementation			

List resources that you will need in order to implement these new strategies or ideas below.

1.
2.
3.
4.
5.

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References

- Carpenter, Thomas, Elizabeth Fennema, Megan L. Franke, Linda Levi, and Susan B. Empson. 1999. *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.
- Coggins, Debra, Drew Kravin, Grace Dávila Coates, and Maria Dreuz Carroll. 2007. *English Language Learners in the Mathematics Classroom.* Thousand Oaks, CA: Corwin Press.
- Daro, Phil. 2011. "Formative Principles of the Common Core Standards." Strategic Education Research Project (SERP), 15 min., 21 sec. Accessed April 7, 2014. http://serpmedia.org/daro-talks/index.html.
- Hiebert, J., Thomas P. Carpenter, Elizabeth Fennema, Karen C. Fuson, Diana Wearne, Hanlie Murray, Piet Human, and Alywn Olivier. 1997. *Making Sense: Teaching and Learning Mathematics with Understanding*. Portsmouth, NH: Heinemann.
- Hull, Ted, Don Balka, and Ruth Harbin Miles. 2011. "Standards of Student Practice in Mathematics Proficiency Matrix." Accessed May 25, 2012. http://www.mathleadership.com/sitebuildercontent/sitebuilderfiles/standardsoftudentpracticeinmathematicsproficiencymatrix.pdf.
- National Council of Teachers of Mathematics (NCTM). 2014. "Principles to Actions: Executive Summary." Accessed July 21, 2015. https://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf
- National Governors Association Center for Best Practices (NGA Center), Council of Chief State School Officers (CCSSO). 2010. "Common Core State Standards for Mathematics." Accessed April 27, 2014. http://www.corestandards.org/Math/.
- Pearson Education, Inc. 2011a. "Problem-Based Interactive Learning." Accessed May 28, 2012. MP4.
- ———. 2011b. "Randall Charles on Mathematical Practices." 6 min., 7 sec. MP4.
- ———. 2011c. "Jane Schielack on Mathematical Practices." 2 min., 12 sec. MP4.
- Public Broadcasting Service (PBS). 2002. "PBS Teacherline: Developing Mathematical Thinking with Effective Questions." Accessed May 24, 2012. http://www.pbs.org/teachers/_files/pdf/TL_MathCard.pdf.
- Small, Marian. 2012. *Good Questions: Great Ways to Differentiate Mathematics Instruction.* 2nd ed. New York, NY: NCTM, Teachers College Press and Nelson Education.
- Van de Walle, John A, Karen Karp, and Jennifer Bay-Williams. 2010*. Elementary and Middle School Mathematics: Teaching Developmentally*, 7th ed. Boston, MA: Allyn & Bacon.
- Van de Walle, John A, and LouAnn Lovin. 2006. *Teaching Student-Centered Mathematics: Grades 3–5*. Boston, MA: Allyn & Bacon.

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Appendix: A Sample Proficiency Matrix for Assessing the Standards for Mathematical Practice

Mathematical Practice	Novice	Intermediate	Expert
Make sense of problems.	Explain the thought process in solving a problem one way.	Explain the thought process in solving a problem and representing it in several ways.	Discuss, explain, and demonstrate solving a problem with multiple representations and in multiple ways.
Persevere in solving them.	Stay with a challenging problem for more than one attempt.	Try several approaches in finding a solution, and only seek hints if stuck.	Struggle with various attempts over time, and learn from previous solution attempts.
Reason abstractly and quantitatively.	Reason with models or pictorial representations to solve problems.	Translate situations into symbols for solving problems.	Convert situations into symbols to appropriately solve problems as well as convert symbols into meaningful situations.
Construct viable arguments.	Explain the thinking for the solution found.	Explain your own thinking and the thinking of others with accurate vocabulary.	Justify and explain, with accurate language and vocabulary, why the solution is correct.
Critique the reasoning of others.	Understand and discuss other ideas and approaches.	Explain other students' solutions, and identify strengths and weaknesses of the solutions.	Compare and contrast various solution strategies, and explain the reasoning of others.
Model with mathematics.	Use models to represent and solve a problem, and translate the solution into mathematical symbols.	Use models and symbols to represent and solve a problem, and accurately explain the solution representation.	Use a variety of models, symbolic representations, and technology tools to demonstrate a solution to a problem.
Use appropriate tools strategically.	Use the appropriate tool to find a solution.	Select from a variety of tools the ones that you can use to solve a problem, and explain the reasoning for the selection.	Combine various tools, including technology, explore and solve a problem, and justify the tool selection and problem solution.
Attend to precision.	Communicate your reasoning and solution to others.	Incorporate appropriate vocabulary and symbols in communicating the solution to others.	Use appropriate symbols, vocabulary, and labeling to effectively communicate and exchange ideas.

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Mathematical Practice	Novice	Intermediate	Expert
Look for and make use of structure.	Look for structure within mathematics to help solve problems efficiently (such as 2 × 7 × 5 has the same value as 2 × 5 × 7, so instead of multiplying 14 × 5, which is (2 × 7) × 5, the student can mentally calculate 10 × 7).	Compose and decompose number situations and relationships through observed patterns in order to simplify solutions.	See complex and complicated mathematical expressions as component parts.
Look for and express regularity in repeated reasoning.	Look for obvious patterns, and use if/then reasoning strategies for obvious patterns.	Find and explain subtle patterns.	Discover deep, underlying relationships. For instance, uncover a model or equation that unifies the various aspects of a problem, such as a discovery of an underlying function.

(NGA Center and CCSSO 2010, 6–8; Hull, Balka, and Harbin Miles 2011)

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Evaluation

enVisionmath2.0

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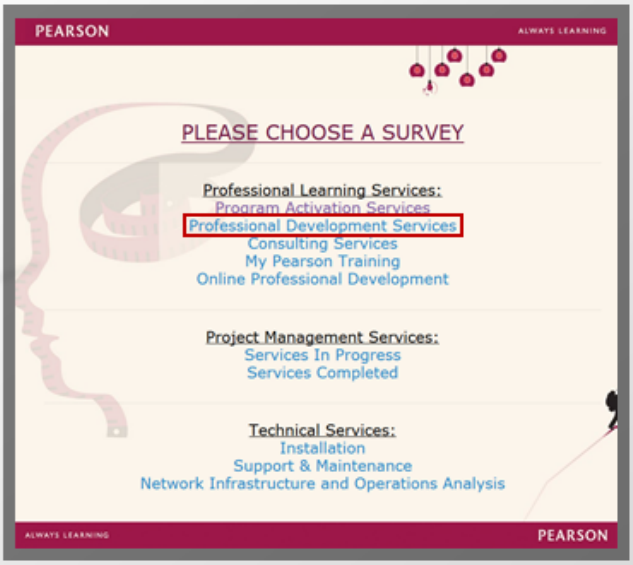

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